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TRANSIENT RESPONSE OF LONG STRUCTURAL MEMBERS.(U)

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Technical Report

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TRANSIENT RESPONSE OF LONG STRUCTURAL MEMBERS

By David Hsu[1] and Walter Pilkey[2], M. ASCE

INTRODUCTION

The dynamic response of long structural members has usually been approached using wave propagation methodology. With such methods an arbitrary structural member, with variations in geometry and material and complicated loading, is a challenge to analyze for transient response. A simple procedure is proposed here which reduces the long member analysis to that of a finite length member analysis. This permits a long member with arbitrary loading, material, and geometry to be studied using the integration techniques commonly employed for finite members.

The proposed procedure first requires the reduction of the governing partial differential equation to an ordinary differential equation in the spatial variable. The time variation can be removed using a finite difference discretization or with a transform. Here, we employ a Laplace transform. The resulting ordinary differential equation in the spatial variable can be solved for a long member using the transfer matrix procedure for infinite beams of Ref. (1). This method will accept members with arbitrary loading and changes in cross-sectional material and geometric properties. The final solution in the time domain is then obtained by inverting the solution in the transform domain. Usually the inverse transform must be computed numerically.

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BASIC THEORY

The procedure will first be illustrated using the longitudinal motion of a bar. The governing equations of motion for the axial extension of a bar are (2)

$$\begin{aligned}\frac{\partial u}{\partial x} &= \frac{P}{AE} + \alpha\Delta T \\ \frac{\partial P}{\partial x} &= \rho\,\frac{\partial^2 u}{\partial t^2} - p_x(x,t) + k_x u\end{aligned} \qquad (1)$$

in which u = Axial displacement, P = axial force, E = modulus of elasticity, A = cross-sectional area, ρ = mass/length, p_x = distributed axial force, α = coefficient of thermal expansion, ΔT = change of temperature, k_x = elastic foundation modulus. To simplify the equations, ignore the $\alpha\Delta T$, and $k_x u$ terms. Then Eqs. (1) can be combined to give the second order equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2}\,\frac{\partial^2 u}{\partial t^2} - \frac{p_x(x,t)}{EA} \qquad (2)$$

where $c = \sqrt{EA/\rho}$.

Consider a very long bar, such as a semi-infinite bar ($x \geq 0$). We shall assume that $p_x(x,t) = 0$ for $x > a$ and that all loadings, changes of materials, variations of cross-sectional area, etc. occur only to the left of $x = a$. In other words, we are treating Eq. (2) in two parts:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{{c_i}^2}\,\frac{\partial^2 u}{\partial t^2} - \frac{p_{ix}(x,t)}{E_i A_i} \qquad a_i > x > a_{i-1} \qquad i = 1, 2, \ldots, n \qquad (3)$$

where $0 = a_o < a_1 < a_2 < \ldots < a_n = a$

and

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\,\frac{\partial^2 u}{\partial t^2} \qquad x > a \qquad (4)$$

To solve Eqs. (3) and (4) with initial conditions and boundary conditions,

use the Laplace transformation on the independent variable t, so that the partial differential equations are reduced to the ordinary differential equations

$$\frac{d^2\bar{u}}{dx^2} = \frac{1}{c_i^2} s^2\bar{u} - su(x,0) - \dot{u}(x,0) - \frac{\bar{P}_{ix}}{E_i A_i} \qquad a_i \geq x \geq a_{a-1}, \quad i = 1,\ldots,n \tag{5}$$

$$\frac{d^2\bar{u}}{dx^2} = \frac{1}{c^2} s^2\bar{u} - su(x,0) - \dot{u}(x,0) \qquad x > a \tag{6}$$

where $\bar{u}$ = Laplace transform of u, $\bar{P}$ = Laplace transform of P. The left-end boundary condition at x = 0 is given, while the boundary condition at the infinite end requires a finite value of $\bar{u}$ for $x = \infty$.

For $u(x,0) = \dot{u}(x,0) = 0$, Eq. (6) has the solution

$$\bar{u}(x,s) = \alpha e^{-\frac{s}{c}(x-a)} + \beta e^{\frac{s}{c}(x-a)} \tag{7}$$

and the bounded response requirement at infinity demands that $\beta = 0$. Define the state vector $z(x) = [\bar{u}\ \bar{P}\ 1]^T$. Also, define $C = [\alpha\ 0\ \ 1]^T$. For $x > a$, note that

$$z(x) = \begin{bmatrix} e^{-\frac{s}{c}(x-a)} & e^{\frac{s}{c}(x-a)} & 0 \\ -\frac{sEA}{c} e^{-\frac{s}{c}(x-a)} & \frac{sEA}{c} e^{\frac{s}{c}(x-a)} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ 0 \\ 1 \end{bmatrix} = Q(x,a)C \tag{8}$$

To compute C in terms of the state vector at a, substitute x = a in Eq. (8) and form

$$C = \begin{bmatrix} \frac{1}{2} & -\frac{c}{2sEA} & 0 \\ \frac{1}{2} & \frac{c}{2sEA} & 0 \\ 0 & 0 & 1 \end{bmatrix} z(a) = Q(a,a)^{-1} z(a) \tag{9}$$

For $x \leq a$, use the usual transfer matrix method

$$z(x) = U_i(x,a_{i-1})\, U_{i-1}(a_{i-1},a_{i-2}) \ldots U_1(a_1,0)\, z(0) \qquad a_i > x > a_{i-1} \tag{10}$$

with U_i being the transfer matrices in $[a_{i-1},a_i]$ obtained from Eqs. (5). Use of the transfer matrix method permits the inclusion of arbitrary loadings and variations in material and geometry.

One of the state variables at $x=0$ ($\bar{u}(0,s)$ or $\bar{P}(0,s)$) is known from the boundary condition at $x=0$. The other state variable at $x=0$ can be determined by equating $z(a) = U_n(a,a_{n-1})\, U_{n-1}(a_{n-1},a_{n-2}) \ldots U_1(a_1,0)z(0) \overset{\Delta}{=} U(a,0)z(0)$ and $z(a) = Q(a,a)C$, i.e,

$$U(a,0)z(0) = Q(a,a)C \tag{11}$$

This expression fixes α and then is used to evaluate the unknown state variable at $x = 0$. Now that both $\bar{u}(0,s)$ and $\bar{P}(0,s)$ are known, i.e., $z(0)$ has been found, the state vector $z(x)$ can be calculated using

for $a_i \geq x \geq a_{i-1} \quad i = 1,2,\ldots,n$

$$z(x) = U_i(x,a_{i-1})U_{i-1}(a_{i-1},a_{i-2}) \ldots U_1(a_1,0)z(0) \tag{12}$$

for $x \geq a$

$$z(x) = Q(x,a)C = Q(x,a)Q^{-1}(a,a)z(a) = Q(x,a)Q^{-1}(a,a)U(a,0)z(0) \tag{13}$$

The solution $\bar{u}(x,s)$ is then transformed back to the time domain through the formula

$$u(x,t) = \frac{1}{2\pi i}\int_{\Gamma} \bar{u}(x,s)e^{st}ds \tag{14}$$

If the integral of Eq. (14) is too difficult to evaluate analytically, the discrete fast Fourier transform routine (3) can be used for a numerical, approximate evaluation.

If the bar is semi-infinite to the left ($x \leq 0$), α rather than β should be set to 0, and Eq.(11) would be replaced by

$$U(b,0)z(0) = Q(b,b)D \text{ with } D = [0 \quad \beta \quad 1]^T \tag{15}$$

Here b is the point to the left of which no variation in material, geometry, or loading can occur. For bars which are infinite at both ends, Eq. (15) is changed to

$$Q(a,a)C = U(a,b)Q(b,b)D \tag{16}$$

from which we determine α in C and β in D, or, equivalently, the state vector at $x=0$.

BEAMS

For the dynamic response of beams, the 4th order governing homogeneous equation after the Laplace transformation is

$$\frac{d^4\bar{u}}{dx^4} + (\zeta - \eta)\frac{d^2\bar{u}}{dx^2} + (\lambda - \zeta\eta)\bar{u} = 0 \tag{17}$$

where $\zeta = (P - \rho r^2 s^2 - k^*)/EI$, $\eta = (k + \rho s^2)/GA_s$, $\lambda = (k + \rho s^2)/EI$

The initial conditions are taken to be zero in Eq. (17). The nomenclature of Ref. (2) has been used. The quantities ζ, η, and λ are complex because of the presence of s. Let $\bar{u}(x,s) = e^{-rx}$ and substitute it into Eq. (17).

$$r^4 + (\zeta - \eta)r^2 + (\lambda - \zeta\eta) = 0 \tag{18}$$

Since the coefficients are complex, denote

$$p + qi = (\zeta - \eta)^2 - 4(\lambda - \zeta\eta), \quad a + bi = (\zeta - \eta) \tag{19}$$

then $r^2 = [-(a + bi) \pm (e + fi)]/2$

where $e + fi = e + qi/(2e)$, $e = \sqrt{(p + \sqrt{p^2 + q^2})/2}$

The square root of r^2 gives the four roots of Eq. (18)

$$\alpha_1 = (h_1 + k_1 i),\quad \alpha_2 = (h_2 + k_2 i),\quad \alpha_3 = -\alpha_1,\ \alpha_4 = -\alpha_2$$

with

$$h_1 = \sqrt{\frac{(-a + e) + \sqrt{(-a + e)^2 + (-b + f)^2}}{2}},\quad k_1 = \frac{(-b + f)}{2h_1}$$

$$h_2 = \sqrt{\frac{(-a - e) + \sqrt{(-a - e)^2 + (-b - f)^2}}{2}},\quad k_2 = \frac{(-b - f)}{2h_2}$$

Hence the general solution for Eq. (17) is

$$\bar{u}(x,s) = C_1 e^{\alpha_1 (x-a)} + C_2 e^{\alpha_2 (x-a)} + C_3 e^{-\alpha_1 (x-a)} + C_4 e^{-\alpha_2 (x-a)} \qquad (20)$$

Since $h_1 > 0$, $h_2 > 0$ for any value of s, C_1 and C_2 must be set to zero in order for $\bar{u}$ to be bounded at $x = \infty$. It is readily shown that for the state vector $z(x) = [\bar{u}\ \ \bar{\theta}\ \ \bar{M}\ \ \bar{V}]^T$ and $C = [C_1\ C_2\ C_3\ C_4]^T$

Here

$$Q(a,a) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\gamma}(\frac{\eta}{GA_s} - \frac{1}{EA}) & 0 & -\frac{1}{GA_s}\gamma & 0 \\ EI\eta & 0 & -EI & 0 & 0 \\ 0 & \frac{1}{\gamma}(\eta - \zeta) & 0 & -\frac{1}{\gamma} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ \alpha_1 & \alpha_2 & -\alpha_1 & -\alpha_2 & 0 \\ \alpha_1^2 & \alpha_2^2 & \alpha_1^2 & \alpha_2^2 & 0 \\ \alpha_1^3 & \alpha_2^3 & -\alpha_1^3 & -\alpha_2^3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (21)$$

The two unknown state variables at $x=0$ can be determined from $U(a,0)z(0) = Q(a,a)C$. The rest of the procedure remains the same as that for the axial motion of bars.

EXAMPLE

As an example of this technique, the stepped semi-infinite bar of Fig. 1 was analyzed. The applied loading is $P(t) = P_o te^{-t}$. The initial conditions are $u(x,0) = 0$, $\dot{u}(x,0) = 0$, the boundary condition at the left end is $P(0,t) = 0$, and $u(\infty,t)$ is bounded.

$$U_1(x,0) = \begin{bmatrix} \cosh \frac{s}{c_1} x & \frac{c_1}{sE_1A_1} \sinh \frac{s}{c_1} x & 0 \\ \frac{sE_1A_1}{c_1} \sinh \frac{s}{c_1} x & \cosh \frac{s}{c_1} x & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad a_1 > x \geq 0 \tag{22}$$

$$U_1^*(a_1^+, a_1^-) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{P_o}{s} \\ 0 & 0 & 0 \end{bmatrix} \qquad x = a_1 \tag{23}$$

The transfer matrix $U_2(x,a_1)$, for $a_2 \geq x \geq a_1$, is obtained from Eq. (22) by replacing x, c_1, E_1, A_1 by $x-a_1$, c_2, E_2, A_2 respectively. The quantity $Q(x,a_2)$, $x \geq a_2$, is taken from $Q(x,a)$ by replacing a, c, E, A by a_2, c_3, E_3, A_3, respectively.

From $U(a_2,0)z(0) = Q(a_2,a_2)C$, the initial parameter $\bar{u}(0,s)$ is found to be

$$\bar{u}(o,s) = \left[-P_o/s \, \frac{E_3A_3c_2}{E_2A_2c_3} \sinh \frac{s}{c_2}(a_2 - a_1) + \cosh \frac{s}{c_2}(a_2 - a_1) \right] \Big/ H \tag{24}$$

$$\text{with } H = \frac{sE_2A_2}{c_2} \sinh \frac{s}{c_2}(a_2 - a_1) + \frac{sE_1A_1}{c_1} \cosh \frac{s}{c_2}(a_2 - a_1) \sinh \frac{s}{c_1} a_1$$

$$+ \frac{sE_3A_3}{c_3} \cosh \frac{s}{c_2}(a_2 - a_1) \cdot \cosh \frac{s}{c_1} a_1 + \frac{sc_2E_1A_2E_3A_3}{c_1c_3E_2A_2} \sinh \frac{s}{c_2}(a_2 - a_1)$$

For a more complicated problem the evaluation of $\bar{u}(0,s)$ and $\bar{u}(x,s)$ would be handled numerically. The transfer matrix method can now be used to provide $\bar{u}(x,s)$ for any x.

Figure 1 shows the displacement at x = 15 for the bar with $A_1 = 1$, $A_2 = 2$, $A_3 = 3$, $\rho_1 = 0.002$, $\rho_2 = 0.004$, $\rho_3 = 0.006$, $E_1 = E_2 = E_3 = 10^7$, $P_o = 20$, $a_1 = 10$, $a_2 = 20$.

SUMMARY AND CONCLUSIONS

A technique is set forth by which the dynamic response can be found for long structural members with arbitrary axial variations in applied loading, geometry, and material. This simple procedure reduces the long member analysis to that of a member of finite length.

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APPENDIX-REFERENCES

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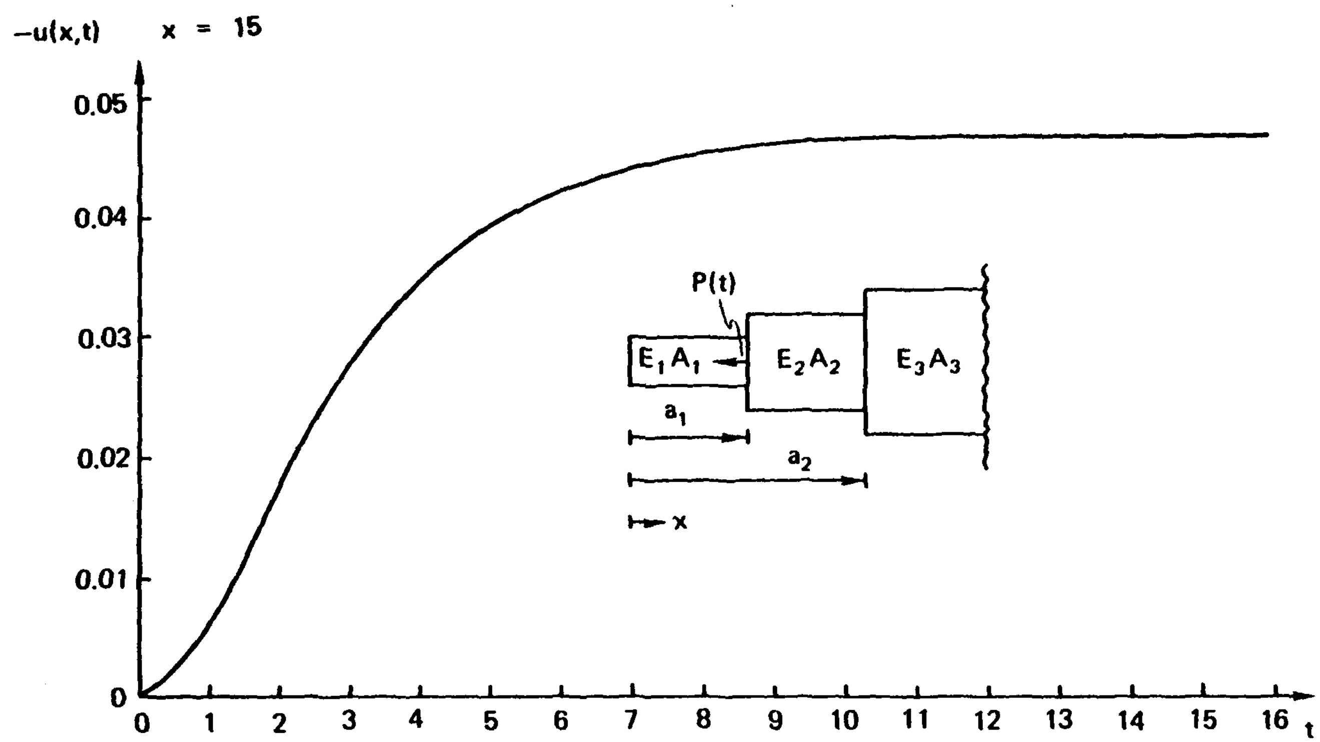

Fig. 1 Semi-infinite bar. Displacement at $x = 15$.

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